Work and Energy

Name: ______________________

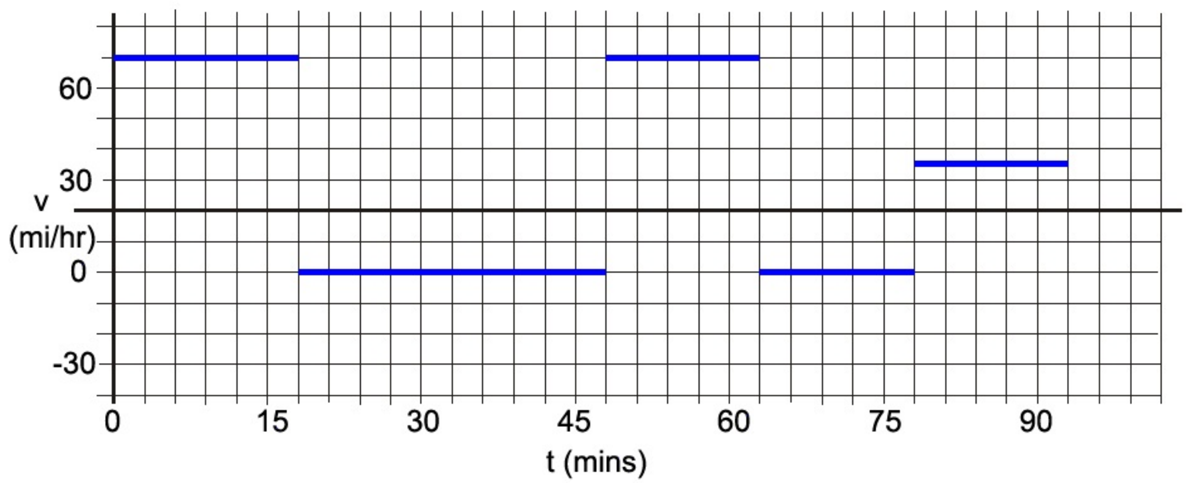
1. A 0.44 kg ball is thrown straight down from a bridge with an initial velocity of 12.5 m/s. It travels for 1.5 seconds before hitting the water below. Find: (a) The height of the bridge, (b) the potential energy of the ball before it is thrown, and (c) the total energy of the ball 2.50 m above the water below.

a. $d_y = d_{yi} + v_i t + \frac{1}{2} a t^2$
$= 0\text{ m} + 12.5\text{ m/s}(1.5\text{ s}) + \frac{1}{2}(9.8\text{ m/s}^2)(1.5\text{ s})^2 = 29.775\text{ m} = \boxed{30.\text{ m}}$

b. $PE = mgh = 0.44\text{ kg} \cdot 9.8\text{ m/s}^2 \cdot 29.775\text{ m} = 128.3898\text{ J} = \boxed{130\text{ J}}$

c. $\sum E$ (at any point) = $\sum E_i$ (as long as no energy is "lost" to friction/heat)
$\sum E$ (2.50 m above water) $= PE_i + KE_i = mgh + \frac{1}{2} mv^2$
$= 128.3898\text{ J} + 0.5 \cdot 0.44\text{ kg} \cdot (12.5\text{ m/s})^2 = 162.7648\text{ J} = \boxed{160\text{ J}}$

2. You travel down the highway, starting from rest. You travel for 0.30 hours at a speed of 70 mi/h. Then you stop and eat your lunch for 30.0 min. Then you travel for 0.25 hours at 70 mi/h. Then you are forced to wait for 15 minutes for roadwork. Then you travel for 15 minutes at only 35 mi/h. Make a velocity vs time graph of this motion.



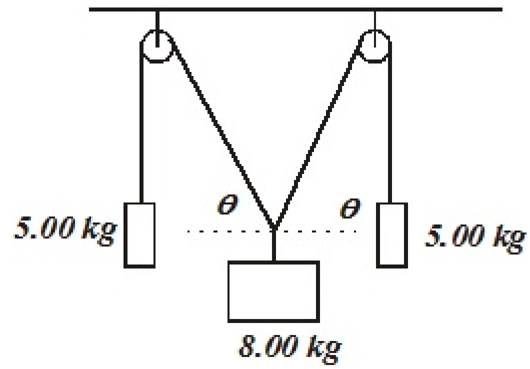
3. A 2.5 kg box slides across the flat surface of a table. The coefficient of kinetic friction for the table/box is 0.295. The box is attached to a light string that passes over a low friction pulley and is connected to a 3.0 kg mass that is hanging vertically. (a) find the acceleration of the system (b) find the velocity of the 2.5 kg box after it has been dragged 0.25 m if its initial velocity was 0.25 m/s, and (c) find the kinetic energy of the box at this point.

a. $F_{net\ sys} = w_{3kg} - F_{fric\ 2.5} = 3.0\text{ kg} \cdot 9.8\text{ m/s}^2 - 0.295 \cdot 2.5\text{ kg} \cdot 9.8\text{ m/s}^2 = 22.1725\text{ N}$
$a_{sys} = F_{net\ sys}/m_{sys} = 22.1725\text{ N} / 5.5\text{ kg} = 4.031363636\text{ m/s}^2 = \boxed{4.0\text{ m/s}^2}$

b. $v^2 = v_i^2 + 2ad = (0.25\text{ m/s})^2 + 2(4.031363636\text{ m/s}^2)(0.25\text{ m}) = 2.078181818\text{ m}^2/\text{s}^2$
$v = 1.441590031\text{ m/s} = \boxed{1.4\text{ m/s}}$

c. $KE = \frac{1}{2} mv^2 = 0.5 \cdot 2.5\text{ kg} \cdot (1.441590031\text{ m/s})^2 = 2.597727272\text{ J} = \boxed{2.6\text{ J}}$

4. Find the two angles if the system is at rest.



No acceleration, so no net force across each pulley.

$T = 5.00\ \text{kg} \cdot 9.8\ \text{m/s}^2 = 49\ \text{N}$

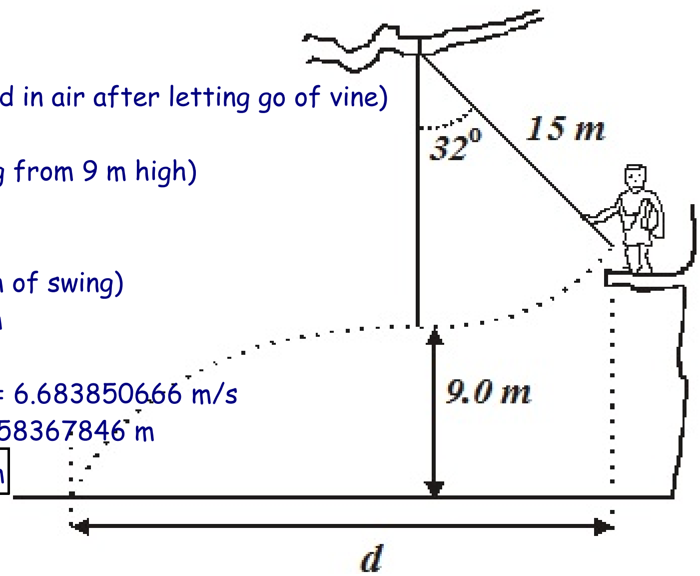
Since the side masses are equal, each side mass will support one half of the 8 kg mass' weight.

$T \cdot \sin\theta = 0.5 \cdot 8.00\ \text{kg} \cdot 9.8\ \text{m/s}^2$

$\sin\theta = (0.5 \cdot 8.00\ \text{kg} \cdot 9.8\ \text{m/s}^2)/49\ \text{N} = 0.8$

$\theta = \sin^{-1}(0.8) = 53.13010235° = \boxed{53.1°}$

5. Okay, here's a wonderful Tarzan swing problem. Tarzan is above the floor of the jungle on a limb. He swings out on a vine and lets go of the thing when he is at the lowest point of the swing. At this point, he is 9.0 m above the ground. How far horizontally did he travel from when he first started his swing?



$d = d_1 + d_2$

($d_1 = d_{horiz}$ from limb to bottom of swing, d_2 = d in air after letting go of vine)

$d_1 = 15\ \text{m} \cdot \sin(32°) = 7.948788963\ \text{m}$

$d_2 = v_{horiz}$ (bottom of swing) $\cdot$ t (in air dropping from 9 m high)

$9.0\ \text{m} = \frac{1}{2}(9.8)t^2$

$t = (9.0\ \text{m} / 4.9\ \text{m/s}^2)^{\frac{1}{2}} = 1.355261854\ \text{s}$

ΔKE (bottom of swing) = ΔPE (limb to bottom of swing)

$\Delta h = 15\ \text{m} - 15\ \text{m} \cdot \cos(32°) = 2.279278558\ \text{m}$

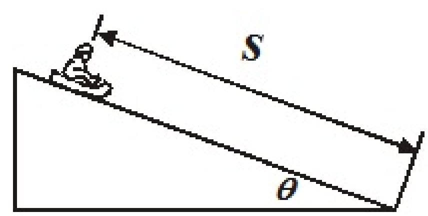
$\frac{1}{2} mv^2 = mgh$

$v = (2gh)^{\frac{1}{2}} = (2 \cdot 9.8\ \text{m/s}^2 \cdot 2.279278558\ \text{m})^{\frac{1}{2}} = 6.683850666\ \text{m/s}$

$d_2 = 6.683850666\ \text{m/s} \cdot 1.355261854\ \text{s} = 9.058367846\ \text{m}$

$d = 7.948788963\ \text{m} + 9.058367846\ \text{m} = \boxed{17\ \text{m}}$

6. A sled coasts down a hill as shown. The angle the slope makes with the horizontal is 41°. The distance ***s*** is 35 m. Find the speed of the sled at the bottom of the hill.

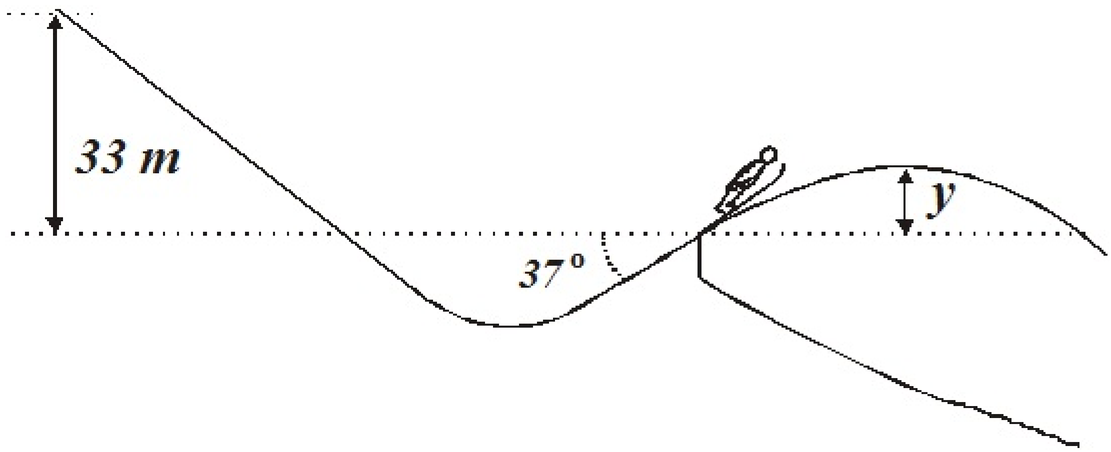


$a_{down\ slope} = 9.8\ \text{m/s}^2 \cdot \sin(41°) = 6.429378484\ \text{m/s}^2$

$v^2 = v_i^2 + 2ad = 0 + 2(6.429378484\ \text{m/s}^2)(35\ \text{m}) = 450.0564939\ \text{m}^2/\text{s}^2$

$v = 21.21453497\ \text{m/s} = \boxed{21\ \text{m/s}}$

7. A ski jumper sails down a slope as shown. Find the vertical distance that the skier travels from the edge of the bottom of the ski jump.



Assuming the skier starts from at rest, $KE_{jump} = PE_{top\ of\ slope}$.

$\frac{1}{2} mv^2 = mgh$

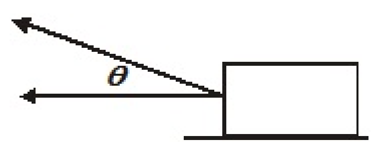
$v = (2gh)^{\frac{1}{2}} = (2 \cdot 9.8\ m/s^2 \cdot 33\ m)^{\frac{1}{2}} = 25.43226297\ m/s$

$v_{vert} = v \cdot \sin(37°) = 15.30551793\ m/s$

$v^2 = v_i^2 + 2ad$

$d = (v^2 - v_i^2)/2a = (0 - 15.30551793^2)/2(-9.8) = 11.95198363\ m = \boxed{12\ m}$

8. You pull a box across the floor with a force of 425 N. The coefficient of kinetic friction is 0.305. The mass of the crate is 125 kg. Angle θ = 35.0°. Find: (a) the acceleration of the box and (b) the amount of work done in moving the crate a distance of 3.50 m.



$F_{up\ pull} = 425\ N \cdot \sin(35.0°) = 243.7699854\ N$

$F_{horiz\ pull} = 425\ N \cdot \cos(35.0°) = 348.1396188\ N$

$F_N = w - F_{up\ pull} = 125\ kg \cdot 9.8\ m/s^2 - 243.7699854\ N = 981.2300146\ N$

a. $F_{net} = F_{pull} - F_{fric} = 348.1396188\ N - 0.305 \cdot 981.2300146\ N = 48.86446435\ N$

$a = F_{net} / m = 48.86446435\ N / 125\ kg = 0.3909157148\ m/s^2 = \boxed{0.391\ m/s^2}$

b. $W = F \cdot d \cdot \cos\theta = 425\ N \cdot 3.50\ m \cdot \cos(35°) = 1218.488666\ J = \boxed{1220\ J}$